See work attached/following this page!!

Math 2 – Linear and Quadratic Systems of Equations WS

Name: Key

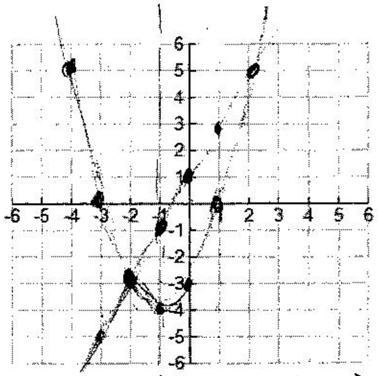
$x^2 - 4x + 4 \quad + 2 - 4$

$\sqrt{(x-2)^2} = \sqrt{-2}$

$x - 2 = \sqrt{-2}$

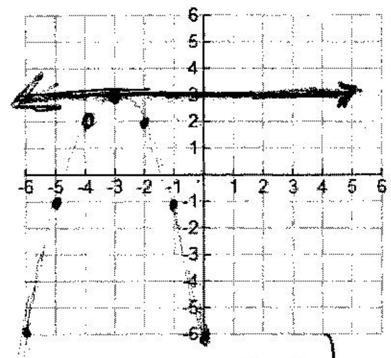
I. Solve each linear and quadratic system BY GRAPHING. State the solution(s) on the line. Must be ACCURATE!

1.) $\begin{cases} y = x^2 + 2x - 3 \\ y = 2x + 1 \end{cases}$



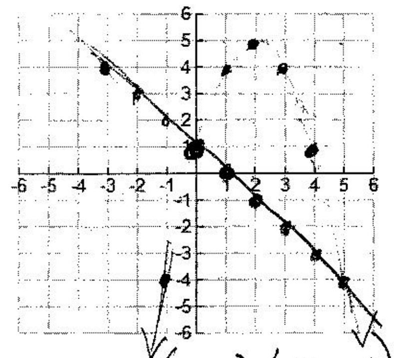
Solution(s): $(-2, -3)$ $(2, 5)$

2.) $\begin{cases} y = -x^2 - 6x - 6 \\ y = 3 \end{cases}$



Solution(s): $(-3, 3)$

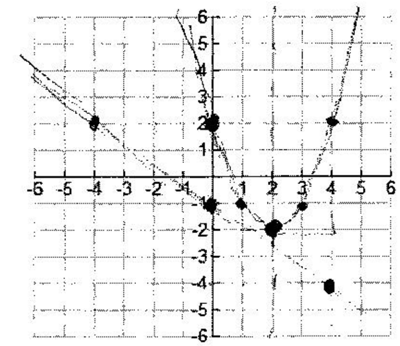
3.) $\begin{cases} y = -(x-2)^2 + 5 \\ y = -x + 1 \end{cases}$



Solution(s): $(0, 1)$ $(5, -4)$

4.) $\begin{cases} y = x^2 - 4x + 2 \\ y = -\frac{3}{4}x - 1 \end{cases}$

vertex $= \frac{-b}{2a}$



Solution(s): No solution

II. Solve each linear and quadratic system BY SUBSTITUTION. State the solution(s) on the line. Must SHOW WORK!

5.) $\begin{cases} y = x^2 + 5x - 2 \\ y = 3x - 2 \end{cases}$ → Solution(s): ______

$(0, -2)$

$(-2, -8)$

6.) $\begin{cases} y = -x^2 - 3x + 2 \\ y = x + 6 \end{cases}$ → Solution(s): $(-2, 4)$

7.) $\begin{cases} y = -2x^2 - 4x - 1 \\ y = 2x + 4 \end{cases}$ → Solution(s): ______

No solution

8.) $\begin{cases} x + y = 5 \\ y + 1 = 3x^2 + 2x \end{cases}$ → Solution(s): ______

$(-2, 7)$

$(1, 4)$

9.) $\begin{cases} x^2 + y - 8 = 0 \\ x + y - 2 = 0 \end{cases}$ → Solution(s): ______

$(3, -1)$

$(-2, 4)$

10.) $\begin{cases} 5x + y = 2x^2 + 6 \\ y + 4x = 7x - 2 \end{cases}$ → Solution(s): ______

$(2, 4)$

Math 2 -- Linear & Quadratic Systems of Equations

#1 $y = x^2 + 2x - 3$

vertex form: $y = a(x-h)^2 + k$

vertex (h, k)

Change to vertex form:

$y = x^2 + 2x \quad -3$ ← Slide this guy out of way for now. Use 1st 2 terms to complete square.

$y = x^2 + 2x + \underline{1} \quad -3 - \underline{1}$

- Since you added 1 as part of process in completing the square, you need to also subtract 1 for balance.

$y = (x+1)^2 - 4$

- Now, the equation is in vertex form.

To Graph w/o calculator WAY 1:

we know the following:

- vertex $(-1, -4)$
- Parabola turns up
- Make same movements as mother function from vertex
- y-intercept: $y = (0+1)^2 - 4$
  $= 1^2 - 4$
  $= -3$
  $(0, -3)$
- x-intercept: $0 = (x+1)^2 - 4$
  $0 = x^2 + 2x + 1 - 4$
  $0 = x^2 + 2x - 3$
  $0 = (x+3)(x-1)$
  $x+3 = 0$ or $x - 1 = 0$
  $x = -3$ or $x = 1$
  $(-3, 0)$ $\quad (1, 0)$

axis of symmetry: $x = -1$

WAY 3 Use calculator.

WAY 2 To find points to graph from standard form $y = x^2 + 2x - 3$

vertex

$x = \frac{-b}{2a} = \frac{-2}{2(1)} = -1$

$y = (-1)^2 + 2(-1) - 3$

$y = 1 - 2 - 3$

$y = -4$

So vertex is at $(-1, -4)$

y-intercept: $y = 0 + 2(0) - 3$

$y = -3$

So y-intercept is at $(0, -3)$

x-intercepts: Set equation equal to 0. Solve for zeros/roots

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x + 3 = 0$ or $x - 1 = 0$

$x = -3$ $\quad x = 1$

$(-3, 0)$ $\quad (1, 0)$

axis of symmetry at $x = -1$

#2) $y = -x^2 - 6x - 6$

To graph w/o calculator

WAY 1: Use standard form to find necessary points.

- vertex: $x = \frac{-b}{2a} = \frac{6}{2(-1)} = \frac{6}{-2} = -3$

  $y = -(-3)^2 - 6(-3) - 6$

  $y = -9 + 18 - 6$

  $y = 3$

  $\boxed{\text{So vertex is at } (-3,3)}$

- y-intercept (when $x = 0$):

  $y = -0^2 - 0 - 6$

  $y = -6$

  $\boxed{(0,-6) \text{ is y-intercept}}$

- x-intercept (when $y = 0$)

  $-x^2 - 6x - 6 = 0$

  No easy factor. Use quadratic formula.

  $a = -1 \quad b = -6 \quad c = -6$

  $x = \frac{6 \pm \sqrt{(-6)^2 - 4(-1)(-6)}}{2(-1)}$

  $x = \frac{6 \pm \sqrt{36 - 24}}{-2}$

  $x = \frac{6 \pm \sqrt{12}}{-2}$

  $x = \frac{6 \pm 2\sqrt{3}}{-2}$

  $x = -3 \pm \sqrt{3}$ → $-3 + \sqrt{3} = -1.268$; $-3 - \sqrt{3} = -4.73$

  ugly points!!

WAY 2: Convert to vertex form

$y = -x^2 - 6x - 6$

$y = -x^2 - 6x \quad -6$

$y = -(x^2 + 6x + 9) - 6 + 9$ complete square

$y = -(x+3)^2 + 3$ vertex form

what do we know:

- Parabola turns down
- $\boxed{\text{Vertex: } (-3,3)}$
- make same movements as mother function -- can find other points from this or...
- y-intercept: (can use either formula really)

  $y = -(0+3)^2 + 3$

  $y = -9 + 3$

  $y = -6$

  $\boxed{\text{So y-intercept } (0,-6)}$

- x-intercept:

  $-(x+3)^2 + 3 = 0$

  $-(x+3)^2 = -3$

  $(x+3)^2 = 3$

  $\sqrt{(x+3)^2} = \pm\sqrt{3}$

  $x + 3 = \pm\sqrt{3}$

  $x = -3 \pm \sqrt{3}$ → $-3 + \sqrt{3} = 1.768$; $-3 - \sqrt{3} = -4.73$

  $(1.768, 0)$ and $(-4.73, 0)$

Axis of symmetry $x = -3$

#3 Graph quadratic

$y = -(x-2)^2+5$ Already in vertex form

- Vertex: $(2,5)$
- Parabola turns down
- SAME Movements as mother function from vertex to get other points OR...

- y-intercept:

$y = -(0-2)^2+5$

$y = -(4)+5$

$y = 1$

$(0,1)$ is y-intercept

X-intercept:

$-(x-2)^2+5=0$ Solve by taking square root

$-(x-2)^2=-5$

$(x-2)^2=5$

$\sqrt{(x-2)^2} = \pm\sqrt{5}$

$x-2=\pm\sqrt{5}$

$x = 2\pm\sqrt{5}$ $\rightarrow 2+\sqrt{5} \approx 4.236$; $\rightarrow 2-\sqrt{5} \approx -0.236$

X-intercepts: $(4.236,0)$ + $(-0.236,0)$

axis of symmetry: $x=2$ (same value as vertex x-value)

4.) Graphic Quadratic

$y = x^2 - 4x + 2$

WAY 1 Graph by finding key points using standard form

Vertex $x\text{-value} = \frac{-b}{2a}$

$= \frac{-(-4)}{2(1)} = \frac{4}{2} = 2$

$y = 2^2 - 4(2) + 2$

$y = 4 - 8 + 2$

$y = -2$

$\boxed{\text{So vertex is at } (2,-2)}$ ← minimum

Axis of symmetry at $x = 2$

y-intercept (when $x = 0$)

$y = 0^2 - 4(0) + 2$

$y = 2$

$\boxed{(0,2) \text{ is y-intercept}}$

X-intercepts (when $y = 0$)

$x^2 - 4x + 2 = 0$

must use quadratic formula or complete square

$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$

$x = \frac{4 \pm \sqrt{16 - 8}}{2}$

$x = \frac{4 \pm \sqrt{8}}{2}$

$x = \frac{4 \pm 2\sqrt{2}}{2}$

$x = 2 \pm \sqrt{2}$ → $2 + \sqrt{2} = 3.41$ → $2 - \sqrt{2} = 0.586$

x-intercepts

$\boxed{(3.41, 0) + (0.586, 0)}$

WAY 2 Complete the square

$y = (x^2 - 4x + \underline{4}) + 2 - \underline{4}$ Keep value same

$\left(\frac{-4}{2}\right)^2 = 4$

$y = (x-2)^2 - 2$ vertex form

$\boxed{\text{So vertex at } (2,-2)}$ ← minimum

- Parabola turns up
- Movements same as mother function
  * can get points by doing

y-intercept: when $x = 0$

$y = (0-2)^2 - 2$

$= 4 - 2$

$= 2$

$\boxed{\text{So y-intercept at } (0,2)}$

x-intercepts; when $y = 0$

$0 = (x-2)^2 - 2$

$(x-2)^2 = 2$

$\sqrt{(x-2)^2} = \pm\sqrt{2}$

$x - 2 = \pm\sqrt{2}$

$x = 2 \pm \sqrt{2}$ → 3.41 → 0.586

$(3.41, 0)$, $(0.586, 0)$ are x-intercepts

axis of symmetry at $x = 2$

Math 2 - Linear and Quadratic Systems of Equations WS

5.) $\begin{cases} y = x^2+5x-2 \\ y = 3x-2 \end{cases}$

To solve by substitution, set equations equal to each other. Solve for x. Plug x-value(s) into an equation to find y.

$x^2+5x-2 = 3x-2$

$x^2+5x-2-3x+2 = 0$

Get all terms to one side. Then you can solve for zeros (x-intercepts or roots).

$x^2+2x = 0$

Simplify

Solve. Pick which method you like best. Factoring works best here.

$x(x+2) = 0$

Product Property of zero

$x=0$ or $x+2=0$

$x=-2$

$x = 0, -2$

Plug these values into either equation to solve for y.

for $x=0$

$y = 3(0)-2$

$y = -2$

So the solution is

$\boxed{(0,-2)}$

for $x=-2$: $y = 3(-2)-2$

$y = -6-2$

$y = -8$

So the solution is

$\boxed{(-2,-8)}$

Check

Eq1 $-2 = 0^2+5(0)-2$

$-2 = -2$ ✓

Eq2 $-2 = 3(0)-2$

$-2 = -2$ ✓

Eq1 $-8 = (-2)^2+5(-2)-2$

$-8 = 4-10-2$

$-8 = -8$ ✓

Eq2 $-8 = 3(-2)-2$

$-8 = -8$ ✓

7.) $\begin{cases} y = -2x^2 - 4x - 1 \\ y = 2x + 4 \end{cases}$

$$-2x^2 - 4x - 1 = 2x + 4$$
$$0 = 2x^4 + 4x + 1 + 2x + 4$$
$$0 = 2x^2 + 6x + 5$$

Solve by quadratic formula or complete square.

Complete square

$$\frac{2x^2}{2} + \frac{6x}{2} = \frac{-5}{2}$$
$$x^2 + 3x + \frac{9}{4} = \frac{-5}{2} + \frac{9}{4}$$
$$\left(\frac{3}{2}\right)^2 = \frac{9}{4}$$
$$\left(x + 3/2\right)^2 = -1/4$$
$$\sqrt{(x + 3/2)^2} = \pm\sqrt{-1/4}$$
$$x + 3/2 = \pm\frac{\sqrt{-1}}{\sqrt{4}}$$
$$x = \frac{-3}{2} \pm \frac{i}{2}$$
$$x = \frac{-3 \pm i}{2}$$

quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 2$ $b = 6$ $c = 5$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(2)(5)}}{2(2)}$$
$$= \frac{-6 \pm \sqrt{36 - 40}}{4}$$
$$x = \frac{-6 \pm \sqrt{-4}}{4}$$

Note imaginary #

$$x = \frac{-6 \pm 2i}{4}$$
$$x = \frac{-3 \pm i}{2}$$

No solution b/c you have imaginary numbers.

wksheet

8. $\begin{cases} X+y=5 \\ y+1=3x^2+2x \end{cases} \Longrightarrow \begin{cases} y=-X+5 \\ y=3x^2+2x-1 \end{cases}$

$$-X+5=3x^2+2x-1$$
$$0=3x^2+2x+x-1-5$$
$$0=3x^2+3x-6$$
$$0=3(x^2+x-2)$$
$$0=3(x+2)(x-1)$$

$x+2=0$ or $x-1=0$

$x=-2$ or $x=1$

Now we need to solve for the y-value that accompanies each x-value.

* using the 1st equation is easiest.

for $x=-2$: $y=-(-2)+5$
$y=7$

$(-2,7)$

For $x=1$: $y=-1+5$
$y=4$

$(1,4)$

So the solutions are $(-2,7)$ and $(1,4)$

(9.) $\begin{cases} x^2+y-8=0 \Rightarrow & y=-x^2+8 \\ x+y-2=0 & y=-x+2 \end{cases}$

$-x^2+8=-x+2$

$0=x^2-x-8+2$

$0=x^2-x-6$

$0=(x-3)(x+2)$

$x-3=0$ or $x+2=0$

$x=3$ $\quad$ $x=-2$

$y=-3+2$ $\quad$ $y=-(-2)+2$

$y=-1$ $\quad$ $y=2+2$

$\quad$ $y=4$

✓ $(3,-1)$ $\quad$ ✓ $(-2,4)$

10.) $\begin{cases} 5x+y=2x^2+6 \Rightarrow & y=2x^2-5x+6 \\ y+4x=7x-2 \Rightarrow & y=7x-4x-2 \end{cases}$

$3x$

$2x^2-5x+6=3x-2$

$2x^2-5x-3x+6+2=0$

$2x^2-8x+8=0$

$2(x^2-4x+4)=0$

$2(x-2)(x-2)=0$

How do I know there's only 1 solution?

$x-2=0$

$x=2$

$(2,4)$ ✓ in both.

$y=3(2)-2$

$y=6-2$

$y=4$