$\qquad$ Class: $\qquad$ Date: $\qquad$

## Module 3 Test Polynomials

## Multiple Choice

Identify the choice that best completes the statement or answers the question.
$\qquad$ 1. Identify the parent function for $g(x)=(x+3)^{3}$ and describe what transformation of the parent function it represents.
A The parent function is the cubic function, $f(x)=x^{3}$. $g(x)=(x+3)^{3}$ represents a vertical translation of the parent function 3 units up.
B The parent function is the cubic function, $f(x)=x^{3}$. $g(x)=(x+3)^{3}$ represents a horizontal translation of the parent function 3 units to the left.
C The parent function is the cubic function, $f(x)=x^{3}$. $g(x)=(x+3)^{3}$ represents a horizontal translation of the parent function 3 units to the right.
D The parent function is the cubic function, $f(x)=x^{3}$. $g(x)=(x+3)^{3}$ represents a vertical translation of the parent function 3 units down.
2. Which is a graph of an even function with a positive leading coefficient?
A

B

C

D

3. Graph $f(x)=x^{3}+6 x^{2}+9 x$. Identify the intercepts and give the domain and range.
A


The $x$-intercepts are -3 and 0 . The $y$-intercept is 0 . The domain and range are all real numbers.

B


The $x$-intercepts are -3 and 0 . The $y$-intercept is 0 . The domain and range are all real numbers.

C


The $x$-intercepts are -1.5 and 0 . The $y$-intercept is 0 . The domain is all real numbers. The range is approximately $y \geq-3.25$.
D


The $x$ - and $y$-intercepts are both zero. The domain and range are all real numbers.
4. If $f(x)$ is an odd function with a negative leading coefficient, $g(x)$ is an even function with a negative leading coefficient, and $h(x)$ is the product of $f(x)$ and $g(x)$, which of the following could be the graph of $h(x)$ ?
A

C

5. Which is the graph of the polynomial function $p(x)=(x-1)(x+1)(x-4)$ ?
A

C

B

D

6. The graph of the polynomial function $p(x)$ is shown. What are the zeros of $p(x)$ ? (Assume that the zeros of $p(x)$ are integers and that the graph of $p(x)$ does not cross the $x$-axis at places other than those shown.)


A $\quad x=-3$ and $x=2$
B $x=-3, x=0$, and $x=2$
C $x=0$
D $x=-2, x=0$, and $x=3$
7. Subtract $2 x y^{3}-3 x y^{3}$.
A $x y^{3}$
C $-x y^{3}$
B $-6 x y^{3}$
D -1
8. Multiply $(x+2)\left(3 x^{2}-4 x+5\right)$.
A $3 x^{3}+2 x^{2}-3 x+10$
C $3 x^{3}-2 x^{2}-3 x+10$
B $3 x^{3}-4 x^{2}+5 x+10$
D $3 x^{3}+10 x^{2}+13 x+10$
9. Multiply $(b+3)\left(b^{2}+2 b+1\right)$.
A $\quad b^{3}+6 b^{2}+6 b+3$
C $3 b^{3}+6 b^{2}+3 b$
B $b^{3}+5 b^{2}+7 b+3$
D $4 b^{3}+8 b^{2}+3 b$
10. Subtract $\left(7 a^{2}-3 a\right)-\left(5 a^{2}-5 a\right)$.
A $2 a^{2}-8 a$
C $\quad 12 a^{2}+2 a$
B $2 a^{2}+2 a$
D $12 a^{2}-8 a$
11. Multiply $(3 x-2)(2 x+6)$.
A $6 x^{2}+5 x+12$
C $6 x^{2}+14 x-12$
B $6 x^{2}-2$
D $6 x^{2}+4$
12. Subtract. $\left(x^{3}-2 x+3\right)-\left(3 x^{2}+4 x-3\right)$
A $-2 x^{3}-6 x+6$
C $x^{3}+3 x^{2}+2 x$
B $-x^{3}+3 x^{2}+6 x-6$
D $x^{3}-3 x^{2}-6 x+6$
13. Find the product.
$\left(x^{2}-2 x-3\right)\left(3 x^{2}+4 x-1\right)$
A $3 x^{4}-2 x^{3}-18 x^{2}-10 x+3$
C $3 x^{4}-x^{3}-18 x^{2}-10 x+3$
B $3 x^{4}-2 x^{3}-16 x^{2}-10 x+3$
D $3 x^{4}-x^{3}-16 x^{2}-10 x+3$
14. Add.
$\left(-7 p^{5} q+6 p q\right)+\left(4 p^{5} q-8 p q+3\right)+(7 p q+7)$
A $-11 p^{5} q+13 p q+10$
C $-3 p^{5} q+5 p q+10$
B $-3 p^{5} q+21 p q+10$
D $-4 p^{5} q+5 p q+9$
15. Multiply.
$(6 r+4 s)^{2}$
A $36 r^{2}+16 s^{2}$
C $36 r^{2}+24 r s+16 s^{2}$
B $\quad 12 r^{2}+8 s^{2}$
D $36 r^{2}+48 r s+16 s^{2}$
16. Which expression represents the perimeter of the triangle below?

A $3+4 m$
C $5+4 m$
B $3+6 m$
D $5+6 m$
17. Which is the product $\left(x^{2}-x+9\right)(x-3)$ ?
A $x^{3}-4 x^{2}+12 x-27$
C $x^{3}+9 x-27$
B $x^{3}-4 x^{2}-27 x+12$
D $x^{3}-3 x^{2}+9 x-9$
18. A rectangular garden has a length of $5 a+17$ feet and a width of $4 a$ feet. Which expression represents the area of the garden in square feet?
A $20 a+68$
C $20 a^{2}+17$
B $20 a^{2}+68 a$
D $25 a^{2}+64 a$
19. Find the product $-2 a^{3} b^{4}\left(3 a^{3} b^{2}+4 b^{4}\right)$.
A $-2 a^{7} b^{7}-2 a^{4} b^{9}$
C $-6 a^{9} b^{8}-8 b^{16}$
B $-6 a^{6} b^{6}-8 a^{3} b^{8}$
D $a^{6} b^{6}+2 a^{3} b^{8}$
20. Find the product of $3 x^{2}+x-1$ and $4 x+5$.
A $3 x^{2}+5 x+4$
C $\quad 12 x^{3}+19 x^{2}+x-5$
B $12 x^{3}+4 x^{2}-4 x$
D $12 x^{3}+19 x^{2}+9 x-5$
21. If $2 x^{2}-5 x+7$ is subtracted from $4 x^{2}+2 x-11$, what is the coefficient of $x$ in the result?

A 2
B 7
C -3
D -18
22. Which divisor of $-2 x^{3}+2 x^{2}-5 x-1$ results in a remainder of 86 ?

A $x+3$
B $x+2$
C $x-2$
D $x-3$
23. Which of the following is a factor of $3 x^{3}-10 x^{2}+3 x+10$ ?

A $x-2$
B $x+3$
C $x-3$
D $x+2$
24. Which of the following is NOT a factor of $\left(x^{3}-x^{2}-14 x+24\right)$ ?
A $x+1$
C $x-3$
B $x-2$
D $x+4$
25. What is the remainder when $2 x^{3}+3 x^{2}-x+1$ is divided by $x+3$ ?
A -25
C -8
B -23
D 25
26. Which is a factor of $x^{3}-3 x^{2}-4 x+12$ ?
A $(x+3)$
C $(x+4)$
B $(x-2)$
D $(x-4)$
27. If $x-2$ is a factor of a polynomial $f(x)$, which of the following statements does NOT have to be true?
A $f(2)=0$
C 2 is a root of $f(x)$.
B $f(-2)=0$
D 2 is a zero of $f(x)$
28. For $p(x)=4 x^{3}-28 x+24, p(-3)=0$. Which of the following must therefore be true?

A -3 is a factor of $p(x)=4 x^{3}-28 x+24$.
B $-3 x$ is a factor of $p(x)=4 x^{3}-28 x+24$.
C $x-3$ is a factor of $p(x)=4 x^{3}-28 x+24$.
D $x+3$ is a factor of $p(x)=4 x^{3}-28 x+24$.
29. Use the remainder theorem to determine the remainder when $p(x)=x^{3}+3 x^{2}-5 x-7$ is divided by $x+5$.

A -182
B -32
C -7
D 168
30. Write an equivalent expression for $a^{2}+2 a b+b^{2}$.
A $\quad(a+b)(a-b)$
C $(a+b)^{2}$
B $a^{2}+b^{2}$
D $a^{2}-b^{2}$
31. Write an equivalent expression for $x^{2}-2 x y+y^{2}$.
A $(x-y)^{2}$
C $(x+y)(x-y)$
B $(x+y)^{2}$
D $x^{2}-y^{2}$
32. Write an equivalent expression for $(a+b)\left(a^{2}-a b+b^{2}\right)$.
A $\quad(a+b)(a-b)^{2}$
C $\quad(a-b)^{3}$
B $a^{3}+b^{3}$
D $a^{3}-b^{3}$
$\qquad$ 33. If you use the polynomial identity $(a+b)(a-b)=a^{2}-b^{2}$ and mental math to calculate $35 \bullet 25$, what subtraction expression results?

A $1000-125$
B $900-25$
C 1050-175
D 1225-625
34. Divide.
$\left(x^{3}+5 x^{2}+5 x-2\right) \div(x+2)$
A $x^{2}-3 x-1$
C $\quad x^{2}+3 x-1$
B $\quad x^{2}-3 x+1$
D $x^{2}+3 x+1$
35. Divide.
$\left(x^{3}-x+6\right) \div(x+2)$
A $\quad x^{2}+3$
C $x^{2}-x+3$
B $\quad x^{2}-2 x+3$
D $(x+1)(x-2)$
36. Divide: $\frac{x^{2}+8 x-5}{x}$
A $x^{2}+8-\frac{5}{x}$
C $x-5$
B $x+8-\frac{5}{x}$
D $x+8$
37. Divide: $\frac{b^{2}-3 b+3}{b-5}$
A $2 b-2+\frac{7}{b-5}$
C $\quad b-8+\frac{49}{b-5}$
B $\quad b+2+\frac{13}{b-5}$
D $\quad b+2+\frac{-13}{b-5}$
38. Divide $12 x^{2}+4 x^{3}+18+16 x$ by $2 x+4$.
A $2 x^{2}+2 x+3+\frac{14}{x+4}$
C $\quad 2 x^{2}+2 x+4-\frac{2}{x+4}$
B $2 x^{2}+2 x+4+\frac{2}{2 x+4}$
D $2 x^{2}+2 x+3+\frac{5}{x+4}$
39. The area of a rectangle is equal to $x^{2}+15 x+44$ square units. If the length of the rectangle is equal to $x+11$ units, what expression represents its width?
A $x-4$
C $x+4$
B $x+33$
D $x-33$
40. Simplify $\frac{x^{3}+4 x^{2}+3 x-2}{x+2}$.
A $(x+1)(x-1)$
C $x^{2}+4 x+3$
B $x^{2}+2 x-1$
D $x^{3}+4 x^{2}+2 x-4$
41. Divide: $\left(5 x+6 x^{3}-8\right) \div(x-2)$.
A $6 x^{2}-12 x+29-\frac{64}{(x-2)}$
C $6 x^{2}+12 x+29+\frac{50}{(x-2)}$
B $6 x^{2}+12 x+29$
D $6 x^{2}+5-\frac{8}{(x-2)}$
42. Write an expression that represents the width of a rectangle with length $x+5$ and area $x^{3}+12 x^{2}+47 x+60$.
A $x^{3}+7 x^{2}+12 x$
C $x^{2}+7 x+12$
B $x^{2}+17 x-38-\frac{50}{x+5}$
D $x^{2}+17 x+132+\frac{720}{x+5}$
43. What is the result if you divide to rewrite the expression $\frac{3 x^{2}-x+7}{x-1}$ ?

A $3 x+\frac{2 x+7}{x-1}$
B $3 x+11$
C $3 x+2$
D $3 x+2+\frac{9}{x-1}$
44. When you divide to simplify the expression $\frac{6 x^{3}+5 x^{2}+2 x+7}{2 x+3}$, what is the fractional part of the quotient?

A $\quad-5$
B $-\frac{5}{2 x+3}$
C $\frac{7}{2 x+3}$
D $3 x^{2}-2 x+4$
45. In the expression $x^{3}+4 x^{2}+3 x+12$, when the first two terms are grouped, and the last two terms are grouped, what is the common binomial factor?
A $x-4$
C $x+4$
B $x^{2}+3$
D $x+12$
46. In the expression $40 x^{2}-15 x+16 x-6$, when the first two terms are grouped, and the last two terms are grouped, what is the common binomial factor?
A $8 x-3$
C $5 x-2$
B $5 x+2$
D $8 x+3$
47. The volume of a box is $20 x^{3}+104 x^{2}+96 x$. What is a possible expression for the height of the box if the width is $2 x$ and the length is $5 x+6$ ?
A $5 x+6$
C $5 x+2$
B $2 x+8$
D $2 x-8$
48. Jon has rewritten the expression $10 x^{3}-35 x^{2}+18 x-63$ in order to factor it. Describe a reasonable next step for Jon to perform.
A Use the Commutative Property to rewrite the terms in a different order.
B Factor 7 from the second and fourth terms.
C Group the first two terms and factor out the greatest common term of $2 x-7$.
D Factor $x$ from each of the four terms.
49. Completely factor $3 x^{4}-15 x^{3}-18 x^{2}$.
A $x^{2}\left(3 x^{2}+2\right)(1 x-9)$
C $3 x^{2}(x+1)(x-6)$
B $3\left(x^{2}+1\right)\left(x^{2}-6\right)$
D cannot be factored
50. What is the complete factorization of $10 x^{3}-35 x^{2}-20 x$ ?
A $(2 x+1)(x-4)$
C $5 x(2 x+1)(x-4)$
B $5 x\left(2 x^{2}-7 x-4\right)$
D $\quad x(2 x+1)(5 x-20)$
51. Completely factor $4 m^{4}-324$.
A $\left(4 m^{2}+36\right)\left(m^{2}-9\right)$
C $\quad 4(m+3)^{2}(m+3)(m-3)$
B $4\left(m^{2}+9\right)(m+3)(m-3)$
D cannot be factored
52. Factor $27 x^{2} z+36 x z+12 z$ completely.
A $z(3 x+12)^{2}$
C $3 z(3 x+2)^{2}$
B $12 z\left(2 x^{2}+3 x+1\right)$
D $3 z(3 x+2)(3 x-2)$
53. Factor $x^{3}+6 x^{2}-25 x-150$.
A $(x-6)(x-5)(x+5)$
C $(x+6)\left(x^{2}+25\right)$
B $(x-6)\left(x^{2}+25\right)$
D $(x+6)(x-5)(x+5)$
54. Factor the expression $54 x^{6}+16 x^{3} y^{3}$.
A $2 x^{3}(3 x+2 y)\left(9 x^{2}+6 x y+4 y^{2}\right)$
C $\quad 2 x^{3}\left(27 x^{3}+8 y^{3}\right)$
B $\quad 2 x^{3}(3 x+2 y)^{3}$
D $2 x^{3}(3 x+2 y)\left(9 x^{2}-6 x y+4 y^{2}\right)$
55. Factor $8 x^{3}+125$.
A $(2 x-5)\left(4 x^{2}-10 x+25\right)$
C $(2 x+5)\left(4 x^{2}-10 x+25\right)$
B $(2 x-5)\left(4 x^{2}+10 x+25\right)$
D $(2 x+5)\left(4 x^{2}+10 x+25\right)$
56. When $x^{3}-125$ is written as a product of a binomial and a trinomial, what is the trinomial factor?

A $x^{2}-5 x+25$
B $x^{2}+5 x+25$
C $x^{2}-10 x+25$
D $x^{2}+10 x+25$
57. Which of the following is equal to $x^{6}-64$ ?

A $-64 x^{6}$
B $\left(x^{3}+8\right)\left(x^{3}-8\right)$
C $\left(x^{3}+8\right)^{2}$
D $\left(x^{3}-8\right)^{2}$
58. A jewelry box has a length that is 2 inches longer than the width and a height that is 1 inch smaller than the width. The volume of the box is 140 cubic inches. What is the width of the jewelry box?
A 2 in.
C 6 in.
B 4 in.
D 5 in.
59. How many roots, including repeated roots, does the function $P(x)=-0.5 x^{3}-1.5 x^{2}+4 x-2$ have?
A 2
B 1
C 0
D 3
60. Which of the following lists all the roots of $x^{3}+3 x=9+3 x^{2}$ ?
A 3
C $3, \pm \sqrt{3} i$
B $3, \pm \sqrt{3}$
D $3, \pm \sqrt{3}, \pm \sqrt{3} i$
61. What are the complex roots of the polynomial $Q(x)=x^{2}+1$ ?
A 1 and -1
C $i$ and $-i$
B $\quad Q(x)$ has no complex roots
D 0 is the only root

## Multiple Response

Identify one or more choices that best complete the statement or answer the question.
$\qquad$ 1. Which of the following statements are true about the polynomial function $p(x)$ ? (The zeros of $p(x)$ are integers, and the graph of $p(x)$ does not cross the $x$-axis at places other than those shown.)


A The degree of $p(x)$ is even.
B The degree of $p(x)$ is 4 .
C The leading coefficient of $p(x)$ is negative.
D The degree of $p(x)$ is at least 6 .
E The graph of $p(x)$ has a $y$-intercept of 150 .
F $\quad p(x)$ has four distinct zeros.
2. Simplify each of the following expressions to determine which are linear.

A $\left(x^{2}+6 x+9\right)+\left(x^{2}-4 x+4\right)$
B $2\left(2 x^{2}+x-10\right)-\left(5 x^{2}-3 x+1\right)$
C $4\left(3 x^{2}+5 x-4\right)-6\left(2 x^{2}+2 x-1\right)$
D $3\left(x^{2}-x+1\right)+\left(-2 x^{2}+4 x-5\right)$
E $\quad 4\left(2 x^{2}-6 x+7\right)-8\left(x^{2}-3 x+4\right)$
3. Use the remainder theorem and the factor theorem to determine which of the following binomials are factors of $p(x)=x^{3}-8 x^{2}+5 x+14$.

A $x+1$
B $x+3$
C $x+9$
D $x-2$
E $x-6$
F $x-7$

## Short Answer

1. Draw a graph of an odd function with exactly two real zeros and a positive leading coefficient.
2. Let $p(x)=x^{3}-2 x^{2}-4 x+8$.
a. Identify the zeros of the function. List all zeros as many times as they occur.
b. Sketch a graph of the function.

3. Let $g(t)=(t+2)\left(t^{2}-5 t+4\right)$.
a. Identify the zeros of the function. Show your work.
b. Sketch a graph of the function.

4. Find the difference $\left(7 a^{3}+5 a\right)-\left(4 a^{3}+4 a\right)$.
5. Multiply $(b-4)\left(b^{2}+3 b-2\right)$.
6. Multiply $(5 x+3 y)^{2}$.
7. Find the product.
$\left(x^{2}-2\right)\left(2 x^{2}+5 x-3\right)$.
8. A triangle has a base of $6 a^{2} c$ and a height of $2 a^{3}-3 a c+1$. Write and simplify an expression for the area of the triangle. Show your work.
9. Use the remainder theorem and the factor theorem to show that $x-5$ is a factor of $p(x)=x^{3}-7 x^{2}+2 x+40$. Then factor $p(x)=x^{3}-7 x^{2}+2 x+40$ completely.
10. Use the remainder theorem to determine the remainder when $p(x)=x^{4}-4 x^{3}-11 x^{2}+66 x-72$ is divided by $x-4$. Then use polynomial long division to verify the remainder.
11. Divide.

$$
\left(x^{3}+x^{2}-20 x+24\right) \div(x-3)
$$

12. Simplify.
$\frac{2 x^{2}-4 x}{x-2}$
13. Divide.
$\left(15 x^{2}+10 x-5\right) \div 5 x$
14. Divide.

$$
\left(12 x^{2}-23 x-24\right) \div(4 x+3)
$$

15. Factor $15 a^{3}+20 a^{2}-6 a-8$ by grouping.
16. Let $p(x)=x^{3}-2 x^{2}-4 x+8$.
a. Identify the zeros of the function. List all zeros as many times as they occur.
b. Sketch a graph of the function.

17. $x^{3}-x^{2}+x-1=0$ is a polynomial equation.

Part A: Explain how you know, without factoring, the number of roots and the minimum number of real roots.

Part B: Factor the polynomial to support your answer to Part A. Explain which factor(s), if any, indicate(s) that there are complex roots.
18. $f(x)=x^{4}-16$ is a polynomial function.

Part A: How many zeros does $f$ have? What are the possible combinations of real and complex zeros?
Part B: Find the zero(s) of $f(x)=x^{4}-16$. Explain how you found your answer(s).
Part C: Let $g(x)=x^{4}+16$. How many real and complex zeros does $g$ have? Explain.

## Essay

1. Is $x-5$ a factor of $3 x^{3}-17 x^{2}+11 x-5$ ? How do you know?
2. Is $x+3$ a factor of $2 x^{3}+4 x^{2}+x-4$ ? How do you know?
