Algebra 2

Name Key ID: 1

Complex Fractions and Solving Rational Equations

Date _____ Period ____

Simplify each expression.

1) $\dfrac{\dfrac{x-2}{25}}{\dfrac{x-2}{x}} = \dfrac{x-2}{25} \cdot \dfrac{x}{x-2} = \boxed{\dfrac{x}{25}}$

2) $\dfrac{\dfrac{x^2}{x+5}}{\dfrac{x}{25}} = \dfrac{x^2}{x+5} \cdot \dfrac{25}{x} = \dfrac{25x^2}{x(x+5)} = \dfrac{x(25x)}{x(x+5)} = \boxed{\dfrac{25x}{x+5}}$

3) $\dfrac{\dfrac{a^2}{5}}{\dfrac{a}{5}} = \dfrac{a^2}{5} \cdot \dfrac{5}{a} = \dfrac{a^2}{a} = \boxed{a}$

4) $\dfrac{\dfrac{m-2}{3m-2} - \dfrac{3m-2}{m^2}}{\dfrac{m-2}{3m-2}}$

5) $\dfrac{\dfrac{25}{a+4} - \dfrac{a+4}{a}}{\dfrac{25}{a}}$

6) $\dfrac{x^2}{\dfrac{x}{4} + \dfrac{x}{5}}$ (Combine)

$= \dfrac{x^2}{\frac{5}{5}\cdot\frac{x}{4} + \frac{x}{5}\cdot\frac{4}{4}} = \dfrac{x^2}{\frac{5x+4x}{20}}$

$\dfrac{x^2}{\frac{9x}{20}} \Rightarrow \dfrac{x^2 \cdot 20}{9x} = \dfrac{x(20x)}{9x} = \boxed{\dfrac{20x}{9}}$

7) $\dfrac{\dfrac{2x+3}{10} - \dfrac{2}{2x+3}}{\dfrac{5}{x-1} + \dfrac{2x+3}{x-1}}$

8) $\dfrac{\dfrac{2}{3} - \dfrac{1}{2}}{\dfrac{2}{3} + \dfrac{16}{3x}}$

(4) $\dfrac{\frac{m-2}{3m-2} - \frac{3m-2}{m^2}}{\frac{m-2}{3m-2}} = \left(\dfrac{m-2}{3m-2} - \dfrac{3m-2}{m^2}\right) \cdot \dfrac{3m-2}{m-2}$

In this way, I did keep, change, flip first.

$= \dfrac{\cancel{3m-2}}{\cancel{m-2}} \cdot \dfrac{\cancel{m-2}}{\cancel{3m-2}} - \dfrac{3m-2}{m^2} \cdot \dfrac{3m-2}{m-2}$

Remember, you need to have like terms to add or subtract

$\dfrac{m^2(m-2)}{m^2(m-2)} - \dfrac{(3m-2)(3m-2)}{m^2(m-2)}$

$= \dfrac{m^2(m-2) - (3m-2)(3m-2)}{m^2(m-2)}$

$= \dfrac{m^3 - 2m^2 - (9m^2 - 12m + 4)}{m^2(m-2)}$

$= \dfrac{m^3 - 2m^2 - 9m^2 + 12m - 4}{m^2(m-2)}$

Factor Top by grouping To see if anything else can cancel

$= \dfrac{m^3 - 11m^2 + 12m - 4}{m^2(m-2)}$ ← okay to quit here

↑

$= \dfrac{m^2(m-11) + 4(3m-1)}{m^2(m-2)}$ ← NOPE CANNOT DO ANYTHING ELSE

4. $\dfrac{\frac{m-2}{m^2} - \frac{3m-2}{m^2}\cdot\frac{(3m-2)}{(3m-2)}}{\frac{m-2}{3m-2}}$

In this case, I simplified the numerator first.

$$\frac{\frac{m^2(m-2)-(3m-2)(3m-2)}{m^2(3m-2)}}{\frac{m-2}{3m-2}} = \frac{\frac{m^3-2m^2-(9m^2-12m+4)}{m^2(3m-2)}}{\frac{m-2}{3m-2}}$$

$$= \frac{m^3-2m^2-9m^2+12m-4}{m^2(3m-2)} \div \frac{m-2}{3m-2}$$

Now, change, keep, flip

$$= \frac{m^3-11m^2+12m-4}{m^2(3m-2)} \cdot \frac{3m-2}{m-2}$$

$$= \frac{m^3-11m^2+12m-4}{m^2(m-2)}$$

$$= \frac{m^3-11m^2+12m-4}{m^3-2m^2}$$

5. $\dfrac{\frac{25}{a+4}-\frac{a+4}{a}}{\frac{25}{a}}$

WAY 1

$=\left(\frac{25}{a+4}-\frac{a+4}{a}\right)\cdot\frac{a}{25}$

① Keep, change, flip.
② Distribute

$=\frac{a}{\cancel{25}}\cdot\frac{\cancel{25}}{a+4}-\frac{a+4}{\cancel{a}}\cdot\frac{\cancel{a}}{25}$

$=\frac{a}{a+4}-\frac{a+4}{25}$

③ Need common denominator to combine

LCD: $25(a+4)$

$=\frac{25}{25}\cdot\frac{a}{a+4}-\frac{a+4}{25}\cdot\frac{(a+4)}{(a+4)}$

$=\frac{25a-(a+4)(a+4)}{25(a+4)}$

$=\frac{25a-(a^2+8a+16)}{25(a+4)}$

$=\frac{25a-a^2-8a-16}{25(a+4)}$

$=\frac{-a^2+17a-16}{25(a+4)}$

$=\frac{-a^2+17a-16}{25a+100}$

$\Rightarrow\frac{-(a^2-17a+16)}{25(a+4)}$

$\Rightarrow\frac{-(a-16)(a-1)}{25(a+4)}$

WAY 2

① $\dfrac{\frac{a}{a}\cdot\frac{25}{a+4}-\frac{a+4}{a}\cdot\frac{(a+4)}{(a+4)}}{\frac{25}{a}}$

② $\dfrac{\frac{25a-(a+4)(a+4)}{a(a+4)}}{\frac{25}{a}}$

③ $\dfrac{\frac{25a-(a^2+8a+16)}{a(a+4)}}{\frac{25}{a}}$

④ $\dfrac{25a-a^2-8a-16}{\cancel{a}(a+4)}\cdot\dfrac{\cancel{a}}{25}$

⑤ $\dfrac{-a^2+17a-16}{25(a+4)}$ or $\dfrac{-a^2+17a-16}{25a+100}$

7. $$\frac{\frac{2x+3}{10} - \frac{2}{2x+3}}{\frac{5}{x-1} + \frac{2x+3}{x-1}}$$ ← can already combine

You can do keep, change, flip first or combine terms in numerator 1st. In this case it seems easier to simplify the numerator first.

$$\frac{\frac{(2x+3)}{(2x+3)}\frac{2x+3}{10} - \frac{2}{2x+3}\cdot\frac{10}{10}}{\frac{2x+8}{x-1}}$$

$$\frac{\frac{(2x+3)(2x+3)}{10(2x+3)} - \frac{20}{10(2x+3)}}{\frac{2x+8}{x-1}} \Rightarrow \frac{\frac{4x^2+12x+9-20}{10(2x+3)}}{\frac{2x+8}{x-1}} = \frac{\frac{4x^2+12x-11}{10(2x+3)}}{\frac{2x+8}{x-1}}$$

Now * Keep, Change, Flip

$$\frac{(4x^2+12x-11)}{10(2x+3)} \cdot \frac{(x-1)}{2x+8}$$

$$= \frac{x(4x^2+12x-11) - 1(4x^2+12x-11)}{(20x+30)(2x+8)}$$

$$= \frac{4x^3+12x^2-11x-4x^2-12x+11}{40x^2+160x+60x+240}$$

$$= \frac{4x^3+8x^2-23x+11}{40x^2+220x+240}$$

8.) $\dfrac{\frac{2}{2}\cdot\frac{2}{3}-\frac{1}{2}\cdot\frac{3}{3}}{\frac{x}{x}\cdot\frac{2}{3}+\frac{16}{3x}}$

In this case, it is easier to simplify the numerator and the denominator first.

$$\frac{\frac{4}{6}-\frac{3}{6}}{\frac{2x}{3x}+\frac{16}{3x}} = \frac{\frac{1}{6}}{\frac{2x+16}{3x}}$$

Now keep, change, flip

$$= \frac{1}{6}\cdot\frac{3x}{2x+16}$$

← can factor out a 3 here

$$= \frac{3x}{12x+96}$$

$$= \frac{3x}{3(4x+32)}$$

$$= \frac{x}{4x+32}$$

11.) $\left(\frac{2}{n} = 4 + \frac{1}{n}\right)n$ LCD: n

$\frac{2}{n} \cdot n = 4 \cdot n + \frac{1}{n} \cdot n$

$2 = 4n + 1$

$1 = 4n$

$\boxed{\frac{1}{4} = n}$

CK: $\frac{2}{\frac{1}{4}} = 4 + \frac{1}{\frac{1}{4}}$

$8 = 4 + 4$

$8 = 8$ ✓

12.) $5x\left(\frac{1}{x} = \frac{1}{5x} - 1\right)$ LCD: $5x$

$5x \cdot \frac{1}{x} = 5x \cdot \frac{1}{5x} - 1 \cdot 5x$

$5 = 1 - 5x$

$5x = 1 - 5$

$5x = -4$

$\boxed{x = -\frac{4}{5}}$

CK: $\frac{1}{\frac{-4}{5}} \stackrel{?}{=} \frac{1}{5 \cdot \frac{-4}{5}} - 1$

$-\frac{5}{4} = -\frac{1}{4} - 1$

$-\frac{5}{4} \stackrel{\checkmark}{=} -\frac{5}{4}$

(13)

$4p\left(\frac{1}{4p}+\frac{1}{4}=\frac{4}{p}\right)$ LCD: $4p$

$4p\cdot\frac{1}{4p}+\frac{1}{4}\cdot 4p=\frac{4}{p}\cdot 4p$

$1+p=16$

$\boxed{p=15}$

CK: $\frac{1}{4\cdot 15}+\frac{1}{4}\stackrel{?}{=}\frac{4}{15}$

$\frac{1}{60}+\frac{1}{4}=\frac{4}{15}$

$\frac{1}{60}+\frac{15}{60}=\frac{16}{60}$

$\frac{16}{60}=\frac{16}{60}$ ✓

(14) $4p^2\left(\frac{p-6}{4p^2}=\frac{3}{2p}-\frac{1}{p^2}\right)$ LCD: $4p^2$

$4p^2\cdot\frac{p-6}{4p^2}=\frac{2p}{4p^2}\cdot\frac{3}{2p}-\frac{1}{p^2}\cdot 4p^2$

$p-6=6p-4$

$-6+4=6p-p$

$-2=5p$

$\boxed{-\frac{2}{5}=p}$

CK: $\frac{-\frac{2}{5}-6}{1}\stackrel{?}{=}\frac{3}{2\left(-\frac{2}{5}\right)}-\frac{1}{\left(-\frac{2}{5}\right)^2}$

$4\left(-\frac{2}{5}\right)^2$

$-10\stackrel{\checkmark}{=}-10$

15. $\frac{6}{6x-5} = \frac{1}{6x-5} + \frac{x-3}{12x-10}$ ← Simplify

LCD: $2(6x-5)$

$2(6x-5)\left(\frac{6}{6x-5} = \frac{1}{6x-5} + \frac{x-3}{2(6x-5)}\right)$

$2(6x-5)\cdot\frac{6}{6x-5} = 2(6x-5)\cdot\frac{1}{6x-5} + \frac{x-3}{2(6x-5)}\cdot 2(6x-5)$

$12 = 2 + x - 3$

$12+1 = x$

$\boxed{13 = x}$

CK:

$\frac{6}{6(13)-5} \overset{\checkmark}{=} \frac{1}{6(13)-5} + \frac{13-3}{12(13)-10}$

(16) $\frac{1}{6b-2} + 1 = \frac{2b-2}{3b-1}$

$2(3b-1)\frac{1}{2(3b-1)} + 1 = \frac{2b-2}{3b-1}$ LCD: $2(3b-1)$

$2(3b-1)\cdot\frac{1}{2(3b-1)} + 1\cdot 2(3b-1) = \frac{2b-2}{3b-1}\cdot 2(3b-1)$

$1 + 6b - 2 = 4b - 4$

$6b - 4b = -4 + 1$

$2b = -3$

$b = -3/2$

Ck:

$\frac{1}{6(-3/2)-2} + 1 = \frac{2(-3/2)-2}{3(-3/2)-1}$

True

17.) $b+1\left(\frac{b^2-9b+18}{b+1} = \frac{4b^2-8b-96}{b+1} + b-6\right)$ LCD: $b+1$

$b+1\left(\frac{b^2-9b+18}{b+1}\right) = b+1 \cdot \frac{4b^2-8b-96}{b+1} + (b-6)(b+1)$

$b^2-9b+18 = 4b^2-8b-96 + b^2-5b-6$

$b^2-9b+18 = 5b^2-13b-102$

$0 = 5b^2-b^2-13b+9b-102-18$

$0 = 4b^2-4b-120$

$0 = 4(b^2-b-30)$

$0 = 4(b-6)(b+5)$

$0 = (b-6)(b+5)$

$b-6=0$ or $b+5=0$

$b=6$ or $b=-5$

check both:

$\frac{6^2-9(6)+18}{6+1} = \frac{4\cdot 6^2-8(6)-96}{6+1} + (6-6)$

$\frac{36-54+18}{7} =$

$\frac{0}{7} =$

18.) $\frac{5x+10}{x^3+3x^2} + \frac{x^2+x-2}{x^3+3x^2} = \frac{1}{x}$ $\quad x \neq 0$

$x(x^2+3x)$ $\qquad x(x^2+3x)$

LCD: $x(x^2+3x)$

$$x(x^2+3x)\cdot\frac{5x+10}{x(x^2+3x)} + \frac{x^2+x-2}{x(x^2+3x)}\cdot x(x^2+3x) = \frac{1}{x}\cdot x(x^2+3x)$$

$$5x+10 + x^2+x-2 = x^2+3x$$

$$6x+8 = 3x$$

$$3x = -8$$

$$\boxed{x = -8/3}$$

CK: $\frac{5\left(\frac{-8}{3}\right)+10}{\left[-8/3\right]^2+3\left(-8/3\right)^2} + \frac{(-8/3)^2+(-8/3)-2}{(-8/3)^2+3(-8/3)^2} = \frac{1}{-8/3}$