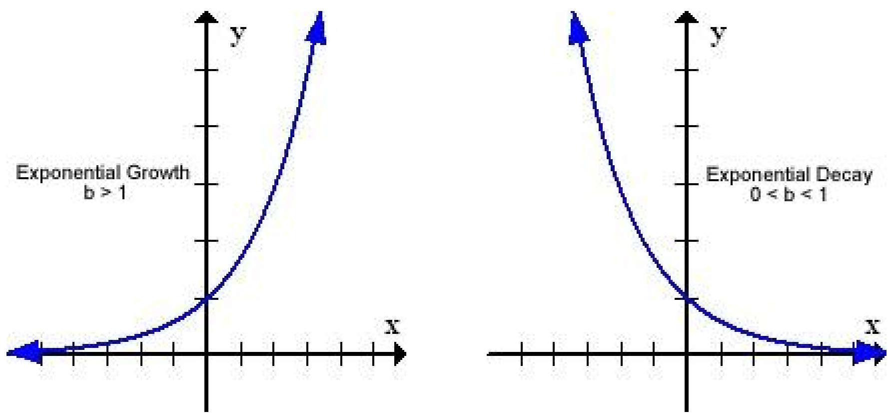
Changing from Exponential to Logarithmic Form

Different Forms?

In the alphabet letters can be written in different ways, either in lower case or upper case and even though the letters may look different the letter still means the same thing. For example, if we take the letter M, it can be written as M or m, but when the letter is used in a word the letter still has the same meaning or sound. The same thing can be done with exponential equations. Exponential equations can be written in logarithmic form and although the equations will look different, the equations still have the same meaning.

What is a Logarithm?

To discuss what a logarithm is, we need to take a look at an exponential function. So, let's start with a generic exponential function, say $y = b^x$. If we draw the graph of the exponential function, we will get one of two possible graphs. If $b > 1$, then the graph created will be exponential growth. If $0 < b < 1$, then the graph created will be exponential decay. Both graphs are shown below.



Looking at the graphs shown above, we can see that no horizontal line can be drawn that would intersect the graph of the exponential functions at more than one point. This means that exponential functions are one-to-one functions and one-to-one functions must have an inverse. So, let's try and find the inverse of an exponential function, $f(x) = b^x$. Here are the steps for finding the inverse function.

Step 1: Does the function have an inverse?	Yes, from above we know that exponential functions are one-to-one functions and one-to-one functions have inverses.
Step 2: Change f(x) to y.	$y = b^x$
Step 3: Switch x and y.	$x = b^y$
Step 4: Solve for y.	This is where the problem arises. Up to this point, we do not have a way to solve this type of problem and this is where logarithms come from. The need to solve the problem $x = b^y$, for y.

Logarithms were created because there needed to be a way to solve the problem $x = b^y$. Otherwise, exponential functions would not have an inverse and since exponential functions are one-to-one they must have an inverse. Basically, logarithmic functions are the inverse of exponential functions.

Definition of Logarithms

Logarithms were created to be the inverse of an exponential function. To give us the ability to solve the problem $x = b^y$ for y. This led to the definition shown below.

> Definition of a Logarithm
>
> For $x > 0$ and $b > 0$, $b \neq 1$, $y = \log_b x$ is equivalent to $b^y = x$.

At first glance this definition does not make a lot of sense, but remember what we said earlier about the letter M, it can be written as M or m and they both mean the same thing. So, let's look at how this idea applies to the definition of a logarithm. The definition of a logarithm gives us the ability to write an equation two different ways and even though these two equations look different, both equations have the same meaning.

From the definition of a logarithm we get two equations, $y = \log_b x$ and $b^y = x$. The equations $y = \log_b x$ and $b^y = x$ are different ways of expressing the same thing. The equation $y = \log_b x$ is written in **logarithmic form** and the equation $b^y = x$ is written in **exponential form**. The two equations are just different ways of writing the same thing, similar to how we can write M two different ways.

> Do not spend too much time trying to understand the meaning of the different equations. All you really need to know is that a logarithmic function is the inverse of an exponential function and all exponential equations can be written in logarithmic form.

Changing from Exponential Form to Logarithmic Form

To learn how to change an equation from exponential form to logarithmic form, let's look at a specific example. Consider the exponential equation $5^x = 25$, what would this equation look like in logarithmic form? The equation $5^x = 25$ would become $x = \log_5 25$, read "x equals log base 5 of 25".

$$5^x = 25 \Rightarrow x = \log_5 25$$

Exponential Form   Logarithmic Form

The key to changing from exponential form into logarithmic form is to pay attention to the "base". In exponential form, $5^x = 25$, the number 5 in this equation is called the "base", the same base as in the logarithmic form of the equation, "log base 5". Notice that the base 5 changes sides, in exponential form the 5 is on the left side of the equal sign, but in logarithmic form the 5 is on the right side of the equal sign. Also note that the x and the 25 in both the exponential form and the logarithmic form did not change sides. The only thing that changes sides is the base 5 and the word "log" is added to the logarithmic equation.

> To change from exponential form to logarithmic form, identify the base of the exponential equation and move the base to the other side of the equal sign and add the word "log". Do not move anything but the base, the other numbers or variables will not change sides.

Examples – Now let's look at some more examples of how to change from exponential form to logarithmic form.

Example 1: Write the exponential equation $4^3 = 64$ in logarithmic form.

$$4^3 = 64 \Rightarrow 3 = \log_4 64$$

In this example, the base is 4 and the base moved from the left side of the exponential equation to the right side of the logarithmic equation and the word "log" was added.

Example 2: Write the exponential equation $6^x = 53$ in logarithmic form.

$$6^x = 53 \Rightarrow x = \log_6 53$$

In this example, the base is 6 and the base moved from the left side of the exponential equation to the right side of the logarithmic equation and the word "log" was added.

Example 3: Write the exponential equation $x^5 = 73$ in logarithmic form.

$$x^5 = 73 \Rightarrow 5 = \log_x 7$$

In this example, the base is x and the base moved from the left side of the exponential equation to the right side of the logarithmic equation and the word "log" was added.

Example 4: Write the exponential equation $98 = 7^y$ in logarithmic form.

$$98 = 7^y \Rightarrow \log_7 98 = y$$

In this example, the base is 7 and the base moved from the right side of the exponential equation to the left side of the logarithmic equation and the word "log" was added.

Example 5: Write the exponential equation $37 = y^5$ in logarithmic form.

$$37 = y^5 \Rightarrow \log_5 37 = y$$

In this example, the base is y and the base moved from the right side of the exponential equation to the left side of the logarithmic equation and the word "log" was added.

Addition Examples

If you would like to see more examples of changing from exponential form to logarithmic form, just click on the link below.

Additional Examples

Practice Problems

Now it is your turn to try a few practice problems on your own. Work on each of the problems below and then click on the link at the end to check your answers.

Problem 1: Write the exponential equation $2^5 = 32$ in logarithmic form.

Problem 2: Write the exponential equation $9^x = 88$ in logarithmic form.

Problem 3: Write the exponential equation $67 = 3^x$ in logarithmic form.

Problem 4: Write the exponential equation $x^6 = 135$ in logarithmic form.

Problem 5: Write the exponential equation $57 = y^4$ in logarithmic form.

Problem 6: Write the exponential equation $7^4 = x$ in logarithmic form.

Solutions to Practice Problems