$$5(2-z)=-1(2+z)$$
$$10-5z=-2-z$$
$$-5z=-12-z$$
$$-4z=-12$$
$$z=3$$

MATHEMATICS CONCEPT REVIEW

This section serves as a review of the mathematical concepts tested on the ACT. Familiarize yourself with the basic mathematical concepts included here and be able to apply them to a variety of math problems.

▶ Pre-Algebra

The fourteen Pre-Algebra (seventh- or eighth-grade level) questions make up about 23% of the total number of questions on the ACT Mathematics Test. The questions test basic algebraic concepts such as:

1. Operations Using Whole Numbers, Fractions, and Decimals
2. Square Roots
3. Exponents
4. Scientific Notation
5. Ratios, Proportions, and Percent
6. Linear Equations with One Variable
7. Absolute Value
8. Simple Probability

Operations Using Whole Numbers, Decimals, and Fractions

The ACT Mathematics Test will require you to add, subtract, multiply, and divide whole numbers, fractions, and decimals. When performing these operations, be sure to keep track of negative signs and line up decimal points in order to eliminate careless mistakes.

The following are some simple rules to keep in mind regarding whole numbers, fractions, and decimals:

1. Ordering is the process of arranging numbers from smallest to greatest or from greatest to smallest. The symbol $>$ is used to represent "greater than," and the symbol $<$ is used to represent "less than." To represent "greater than or equal to," use the symbol $\geq$; to represent "less than or equal to," use the symbol $\leq$.
2. The Commutative Property of Multiplication is expressed as $a \times b = b \times a$, or $ab = ba$.
3. The Distributive Property of Multiplication is expressed as $a(b+c) = ab + ac$.
4. The order of operations for whole numbers can be remembered by using the acronym **PEMDAS**:

   P First, do the operations within the **parentheses**, if any.
   E Next, do the **exponents**.
   MD Next, do the **multiplication** and **division**, in order from left to right.
   AS Finally, do the **addition** and **subtraction**, in order from left to right.

5. When a number is expressed as the product of two or more numbers, it is in factored form. *Factors* are all of the numbers that will divide evenly into one number.
6. A number is called a *multiple* of another number if it can be expressed as the product of that number and a second number. For example, the multiples of 4 are 4, 8, 12, 16, etc., because $4 \times 1 = 4$, $4 \times 2 = 8$, $4 \times 3 = 12$, $4 \times 4 = 16$, etc.
7. The Greatest Common Factor (GCF) is the largest integer that will divide evenly into any two or more integers. The Least Common Multiple (LCM) is the smallest integer into which any two or more integers will divide evenly. For example, the Greatest Common Factor of 24 and 36 is 12, because 12 is the largest integer that will divide evenly into both 24 and 36. The Least Common Multiple of 24 and 36 is 72, because 72 is the smallest integer into which both 24 and 36 will divide evenly.
8. Multiplying and dividing both the numerator and the denominator of a fraction by the same nonzero number will result in an equivalent fraction.
9. When multiplying fractions, multiply the numerators to get the numerator of the product, and multiply the denominators to get the denominator of the product. For example, $\frac{3}{5} \times \frac{7}{8} = \frac{21}{40}$.
10. To divide fractions, multiply the first fraction by the reciprocal of the second fraction. For example, $\frac{1}{3} \div \frac{1}{4} = \frac{1}{3} \times \frac{4}{1}$, which equals $\frac{4}{3}$.
11. When adding and subtracting like fractions, add or subtract the numerators and write the sum or difference over the denominator. So, $\frac{1}{8} + \frac{2}{8} = \frac{3}{8}$, and $\frac{4}{7} - \frac{2}{7} = \frac{2}{7}$.
12. When adding or subtracting unlike fractions, first find the Lowest Common Denominator. The Lowest Common Denominator is the smallest integer into which all of the denominators will divide evenly. For example, to add $\frac{3}{4}$ and $\frac{5}{6}$, find the smallest integer into which both 4 and 6 will divide evenly. That integer is 12, so the Lowest Common Denominator is 12. Multiply $\frac{3}{4}$ by $\frac{3}{3}$ to get $\frac{9}{12}$, and multiply $\frac{5}{6}$ by $\frac{2}{2}$ to get $\frac{10}{12}$. Now add the fractions: $\frac{9}{12} + \frac{10}{12} = \frac{19}{12}$, which can be simplified to $1\frac{7}{12}$.
13. *Place value* refers to the value of a digit in a number relative to its position. Moving left from the decimal point, the values of the digits are 1's, 10's, 100's, etc. Moving right from the decimal point, the values of the digits are 10ths, 100ths, 1000ths, etc.
14. When converting a fraction to a decimal, divide the numerator by the denominator.

Square Roots

A square root is written as $\sqrt{n}$, and is the nonnegative value a that fulfills the expression $a^2 = n$. For example, the square root of 25 would be written as $\sqrt{25}$, which is equivalent to 5^2, or 5×5. A number is considered a perfect square when the square root of that number is a whole number. So, 25 is a perfect square because the square root of 25 is 5.

Exponents

When a whole number is multiplied by itself, the number of times it is multiplied is referred to as the *exponent*. As shown above with square roots, the exponent of 5^2 is 2 and it signifies 5×5. Any number can be raised to any exponential value. For example, $7^6 = 7 \times 7 \times 7 \times 7 \times 7 \times 7 = 117{,}649$.

Scientific Notation

When numbers are very large or very small, scientific notation is used to shorten them. To form the scientific notation of a number, the decimal point is moved until it is placed after the first nonzero digit from the left in the number. For example, 568,000,000 written in scientific notation would be 5.68×10^8, because the decimal point was moved 8 places to the left. Likewise, 0.0000000354 written in scientific notation would be 3.54×10^{-8}, because the decimal point was moved 8 places to the right.

Ratio, Proportion, and Percent

A *ratio* is the relation between two quantities expressed as one divided by the other. For example, if there are 3 blue cars and 5 red cars, the ratio of blue cars to red cars is $\frac{3}{5}$, or 3:5. A *proportion* indicates that one ratio is equal to another ratio. For example, if the ratio of blue cars to red cars is $\frac{3}{5}$, and there are 8 total cars, you could set up a proportion to calculate the percent of blue cars, as follows:

3 cars is to 8 cars as x percent is to 100 percent

$\frac{3}{8} = \frac{x}{100}$; solve for x

$8x = 300$

$x = 37.5\%$

A *percent* is a fraction whose denominator is 100. The fraction $\frac{55}{100}$ is equal to 55%.

Linear Equations with One Variable

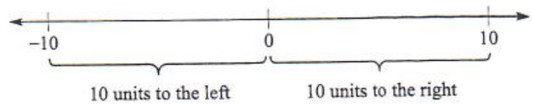
In a linear equation with one variable, the variable cannot have an exponent or be in the denominator of a fraction. An example of a linear equation is $2x + 13 = 43$. The ACT Mathematics Test will most likely require you to solve for x in that equation. Do this by isolating x on the left side of the equation, as follows:

$$2x + 13 = 43$$
$$2x = 43 - 13$$
$$2x = 30$$
$$x = \frac{30}{2}, \text{ or } 15$$

One common ACT example of a linear equation with one variable is in questions involving speed of travel. The basic formula to remember is Rate $\times$ Time = Distance. The question will give you two of these values and you will have to solve for the remaining value.

Absolute Value

The absolute value of a number is notated by placing that number inside two vertical lines. For example, the absolute value of 10 is written as follows: $|10|$. Absolute value can be defined as the numerical value of a real number without regard to its sign. This means that the absolute value of 10, $|10|$, is the same as the absolute value of -10, $|-10|$, in that they both equal 10. Think of it as the distance from -10 to 0 on the number line and the distance from 0 to 10 on the number line: both distances equal 10 units.



Simple Probability

Probability is used to measure how likely an event is to occur. It is always between 0 and 1; an event that will definitely not occur has a probability of 0, whereas an event that will certainly occur has a probability of 1. To determine probability, divide the number of outcomes that fit the conditions of an event by the total number of outcomes. For example, the chance of getting heads when flipping a coin is 1 out of 2, or $\frac{1}{2}$. There are two possible outcomes (heads or tails) but only one outcome (heads) that fits the conditions of the event. Therefore, the probability of the coin toss resulting in heads is 0.5, or 50%.

When two events are independent, meaning the outcome of one event does not affect the other, you can calculate the probability of both occurring by multiplying the probabilities of each of the events together. For example, the probability of flipping three heads in a row would be $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$, or $\frac{1}{8}$. The ACT Mathematics Test will assess your ability to calculate simple probabilities in everyday situations.

▶ Elementary Algebra

The 10 Elementary Algebra (eighth- or ninth-grade level) questions make up about 17% of the total number of questions on the ACT Mathematics Test. The questions test elementary algebraic concepts such as:

1. Functions
2. Polynomial Operations and Factoring Simple Quadratic Expressions
3. Linear Inequalities with One Variable
4. Properties of Integer Exponents and Square Roots

Functions

A function is a set of ordered pairs where no two of the ordered pairs has the same x-value. In a function, each input (x-value) has exactly one output (y-value). An example of this relationship would be $y = x^2$. Here, y is a function of x, because for any value of x there is exactly one value of y. However, x is not a function of y, because for certain values of y there is more than one value of x. The *domain* of a function refers to the x-values, while the *range* of a function refers to the y-values. If the values in the domain corresponded to more than one value in the range, the relation is not a function. The following is an example of a function question that may appear on the ACT Mathematics Test:

For the function $f(x) = x^2 - 3x$, what is the value of $f(5)$?

Solve this problem by substituting 5 for x wherever x appears in the function:

$$f(x) = x^2 - 3x$$
$$f(5) = (5)^2 - (3)(5)$$
$$f(5) = 25 - 15$$
$$f(5) = 10$$

Polynomial Operations and Factoring Simple Quadratic Expressions

A polynomial is the sum or difference of expressions like $2x^2$ and $14x$. The most common polynomial takes the form of a simple quadratic expression, such as $2x^2 + 14x + 8$, with the terms in decreasing order. The standard form of a simple quadratic expression is $ax^2 + bx + c$, where a, b, and c are whole numbers. When the terms include both a number and a variable, such as x, the number is called the *coefficient*. For example, in the expression $2x$, 2 is the coefficient of x.

The ACT Mathematics Test will often require you to evaluate, or solve a polynomial by substituting a given value for the variable, as follows:

For $x = -2$, $2x^2 + 14x + 8 = ?$

$$2(-2)^2 + 14(-2) + 8$$
$$2(4) + (-28) + 8$$
$$8 - 28 + 8$$
$$= -12$$

You will also be required to add, subtract, multiply, and divide polynomials. To add or subtract polynomials, simply combine like terms, as in the following examples:

$$\begin{array}{r} (2x^2 + 14x + \ \ 8) \\ \underline{+(3x^2 + \ \ 5x + 32)} \\ 5x^2 + 19x + 40 \end{array}$$

and

$$\begin{array}{r} (8x^2 + 11x + 23) \\ \underline{-(7x^2 + \ \ 3x + 13)} \\ x^2 + \ \ 8x + 10 \end{array}$$

To multiply polynomials, use the distributive property to multiply each term of one polynomial by each term of the other polynomial. Following are some examples:

$$(3x)(x^2 + 4x - 2) = (3x^3 + 12x^2 - 6x)$$

Remember the **FOIL** Method whenever you see this type of multiplication: multiply the **F**irst terms, then the **O**utside terms, then the **I**nside terms, then the **L**ast terms.

$(2x^2 + 5x)(x - 3) =$
First terms: $(2x^2)(x) = 2x^3$
Outside terms: $(2x^2)(-3) = -6x^2$
Inside terms: $(5x)(x) = 5x^2$
Last terms: $(5x)(-3) = -15x$

Now put the terms in decreasing order:

$$2x^3 + (-6x^2) + 5x^2 + (-15x) = 2x^3 - 1x^2 - 15x$$

You may also be asked to find the factors or solution sets of certain simple quadratic expressions. A factor or solution set takes the form ($x \pm$ some number). Simple quadratic expressions will usually have two of these factors or solution sets. Remember that the standard form of a simple quadratic expression is $ax^2 + bx + c$. To factor the equation, find two numbers that when multiplied together will give you c and when added together will give you b.

The ACT Mathematics Test includes questions similar to the following:

What are the solution sets for $x^2 + 9x + 20$?

Follow these steps to solve:

$x^2 + 9x + 20 = 0$
$(x + \_\_\_)(x + \_\_\_) = 0$

5 and 4 are two numbers that when multiplied together give you 20, and when added together give you 9.

$(x + 5)(x + 4)$ are the two solution sets for $x^2 + 9x + 20$

Linear Inequalities with One Variable

Linear inequalities with one variable are solved in almost the same manner as linear equations with one variable: by isolating the variable on one side of the inequality. Remember, though, that when multiplying one side of an inequality by a negative number, the direction of the sign must be reversed.

The ACT Mathematics Test will include questions similar to those that follow:

For which values of x is $3x + 4 > 2x + 1$?

Follow these steps to solve:

$3x + 4 > 2x + 1$
$3x - 2x > 1 - 4$
$x > -3$

For which values of x is $6x - 32 < 10x + 12$?

Follow these steps to solve:

$6x - 32 < 10x + 12$
$6x - 10x < 32 + 12$
$-4x < 44$

Now, since you have to divide both sides by -4, remember to reverse the inequality sign: $x > -11$.

Properties of Integer Exponents

The ACT Mathematics Test will assess your ability to multiply and divide numbers with exponents. The following are the rules for operations involving exponents:

- $(x^m)(x^n) = x^{(m+n)}$
- $(x^m)^n = x^{mn}$
- $(xy)^m = (x^m)(y^m)$

- $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$
- $x^0 = 1$, when $x \neq 0$
- $x^{-m} = \frac{1}{x^m}$, when $x \neq 0$
- $\frac{a}{x^{-m}} = ax^m$, when $x \neq 0$

▶ Intermediate Algebra

The nine Intermediate Algebra (ninth- or tenth-grade level) questions make up about 15% of the total number of questions on the ACT Mathematics Test. The questions test intermediate algebraic concepts such as:

1. Quadratic Formula
2. Radical and Rational Expressions
3. Inequalities and Absolute Value Equations
4. Sequences
5. Systems of Equations
6. Logarithms
7. Roots of Polynomials

Quadratic Formula

The quadratic formula is expressed as $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$. This formula finds solutions to quadratic equations of the form $ax^2 + bx + c = 0$. It is the method that can be used in place of factoring for more complex polynomial expressions. The quantity $b^2 - 4ac$ is called the *discriminant* and can be used to determine quickly at what kind of answer you should arrive. If the discriminant is 0, then there is only one solution. If the discriminant is positive, then there are two real solutions. If the discriminant is negative, then you will have two complex solutions of the form $(a + bi)$, where a and b are real numbers and i is the imaginary number defined by $i^2 = -1$.

Radical and Rational Expressions

The nth root of a given quantity is indicated by the radical sign, $\sqrt{\ }$. For example, $\sqrt{9}$ is considered a radical, and 9 is the *radicand*. The following rules apply to computations with radical signs:

- $\sqrt{a}$ means the "square root of a," $\sqrt[3]{a}$ means the "cube root of a," etc.
- $\sqrt{a} \times \sqrt{b} = \sqrt{(ab)}$
- $\sqrt[n]{a^n} = a$
- $\sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a}$

A *rational number* is a number that can be expressed as a ratio of two integers. Fractions are rational numbers that represent a part of a whole number. To find the square root of a fraction, simply divide the square root of the numerator by the square root of the denominator. If the denominator is not a perfect square, rationalize the denominator by multiplying both the numerator and the denominator by a number that would make the denominator a perfect square. Consider the following example:

$$\sqrt{\frac{1}{3}} =$$
$$\frac{\sqrt{1}}{\sqrt{3}} = \frac{\sqrt{1}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Inequalities and Absolute Value Equations

An inequality with an absolute value will be in the form of $|ax + b| > c$, or $|ax + b| < c$. To solve $|ax + b| > c$, first drop the absolute value and create two separate inequalities with the word OR between them. To solve $|ax + b| < c$, first drop the absolute value and create two separate inequalities with the word AND between them. To remember this, think of the inequality sign that is being used in the equation. If it is a "greatOR" than sign, use OR. If it is a "less 'thAND'" sign, use AND. The first inequality will look just like the original inequality without the absolute value. For the second inequality, you must switch the inequality sign and change the sign of c.

To solve $|x + 3| > 5$, first drop the absolute value sign and create two separate inequalities with the word OR between them:

$x + 3 > 5$ OR $x + 3 < -5$. Then solve for x:
$x > 2$ OR $x < -8$.

To solve $|x + 3| < 5$, first drop the absolute value sign and create two separate inequalities with the word AND between them:

$x + 3 < 5$ AND $x + 3 > -5$. Then solve for x:
$x < 2$ AND $x > -8$.

Sequences

An arithmetic sequence is one in which the difference between consecutive terms is the same. For example, 2, 4, 6, 8..., is an arithmetic sequence where 2 is the common difference. In an arithmetic sequence, the nth term can be found using the formula $a_n = a_1 + (n - 1)d$, where d is the common difference. A geometric sequence is one in which the ratio between two terms is constant. For example, $\frac{1}{2}$, 1, 2, 4, 8..., is a geometric sequence where 2 is the common ratio. With geometric sequences, you can find the nth term using the formula $a_n = a_1(r)^{n-1}$, where r is the common ratio.

Systems of Equations

The most common type of system of equations question tested on the ACT Mathematics Test involves two equations and two unknowns. Solve this system of equations as follows:

$$4x + 5y = 21$$
$$5x + 10y = 30$$

If you multiply the top equation by -2, you will get:

$$-8x - 10y = -42$$

Now, you can add the like terms of the two equations together, and solve for x:

$$(-8x + 5x) = -3x$$
$$(-10y + 10y) = 0$$

$-42+30=-12$
$-3x=-12$

Notice that the two y-terms cancel each other out. Solving for x, you get $x=4$. Now, choose one of the original two equations, plug 4 in for x, and solve for y:

$4(4)+5y=21$
$16+5y=21$
$5y=5$
$y=1$

Logarithms

Logarithms are used to indicate exponents of certain numbers called *bases*, where $\log_a b=c$, if $a^c=b$. For example, $\log_2 16=4$, which means the log to the base 2 of 16 is 4, because $2^4=16$.

The following is the kind of logarithm problem you are likely to see on the ACT Mathematics Test:

Which of the following is the value of x that satisfies $\log_x 9=2$?

Follow these steps to solve:

$\log_x 9=2$ means the log to the base x of $9=2$.
So, x^2 must equal 9, and x must equal 3.

Roots of Polynomials

When given a quadratic equation, $ax^2+bx+c=0$, you may be asked to find the roots of the equation. This means you need to find what value(s) of x make the equation true. You may either choose to factor the quadratic equation or you may choose to use the quadratic formula. For example, use factoring to find the roots of $x^2+6x+8=0$:

$x^2+6x+8=0$
$(x+4)(x+2)=0$; solve for x
$x+4=0$ and $x+2=0$, so $x=-4$ and $x=-2$

The roots of $x^2+6x+8=0$ are $x=-4$ and $x=-2$. Using the quadratic formula will yield the same solution.

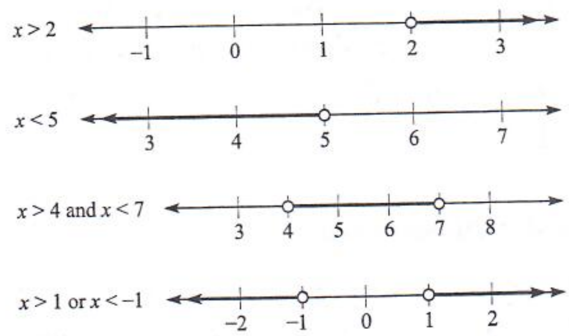
▶ Coordinate Geometry

The nine Coordinate Geometry (Cartesian Coordinate Plane) questions make up about 15% of the total number of questions on the ACT Mathematics Test. The questions test coordinate geometry concepts such as:

1. Number Line Graphs
2. Equation of a Line
3. Slope
4. Parallel and Perpendicular Lines
5. Distance and Midpoint Formulas

Number Line Graphs

The most basic type of graphing is graphing on a number line. For the most part, you will be asked to graph inequalities. Below are four of the most common types of problems you will be asked to graph on the ACT Mathematics Test:



If the inequality sign specifies "greater than or equal to" ($\geq$), or "less than or equal to" ($\leq$), you would use a closed circle instead of an open circle on the designated number or the number line.

Equation of a Line

There are three forms used to write an equation of a line. The standard form of an equation of a line is in the form $Ax+By=C$. This can be transformed into the slope-intercept form of $y=mx+b$, where m is the slope of the line and b is the y-intercept (that is, the point at which the graph of the line crosses the y-axis). The third form is point-slope form, which is $(y-y_1)=m(x-x_1)$, where m is the slope and (x_1, y_1) is a given point on the line. The ACT Mathematics Test will often require you to put the equation of a line into the slope-intercept form to determine either the slope or the y-intercept of a line as follows:

What is the slope of the line given by the equation $3x+7y-16=0$?

Follow these steps to solve:

$3x+7y-16=0$; isolate y on the left side of the equation.

$7y=-3x+16$

$y=-\frac{3}{7}x+\frac{16}{7}$

The slope of the line is $-\frac{3}{7}$.

Slope

The slope of a line is the grade at which the line increases or decreases. Commonly defined as "rise over run," the slope is a value that is calculated by taking the change in y-coordinates divided by the change in x-coordinates for any two given points on a line. The formula for slope is $m=\frac{(y_2-y_1)}{(x_2-x_1)}$ where (x_1, y_1) and (x_2, y_2) are the two given points. For example, if you are given (3,2) and (5,6) as two points on a line, the slope would be $m=\frac{6-2}{5-3}=\frac{4}{2}=2$. A positive slope means the graph of the line will go up and to the right.

A negative slope means the graph of the line will go down and to the right. A horizontal line has slope 0, and a vertical line has undefined slope.

Parallel and Perpendicular Lines

Two lines are parallel if and only if they have the same slope. Two lines are perpendicular if and only if the slope of either of the lines is the negative reciprocal of the slope of the other line. To illustrate, if the slope of line a is 5, then the slope of line b must be $-\frac{1}{5}$ for lines a and b to be perpendicular.

Distance and Midpoint Formulas

To find the distance between two points on a coordinate graph, use the formula $\sqrt{([x_2 - x_1]^2 + [y_2 - y_1]^2)}$, where (x_1, y_1) and (x_2, y_2) are the two given points. For instance, the distance between (3,2) and (7,6) is $\sqrt{([7-3]^2 + [6-2]^2)} = \sqrt{4^2 + 4^2} = \sqrt{(16+16)} = \sqrt{(32)} = \sqrt{(16)(2)} = 4\sqrt{2}$.

Note: This formula is based on the Pythagorean Theorem and if you can't remember it on test day, just draw a right triangle on your test booklet and proceed from there.

To find the midpoint of a line given two points on the line, use the formula $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$. For example, the midpoint between (5,4) and (9,2) is $\left(\frac{5+9}{2}, \frac{4+2}{2}\right) = (7,3)$.

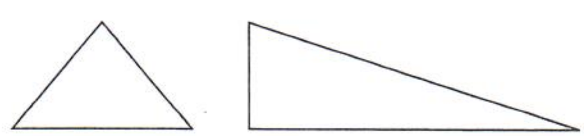
▶ Plane Geometry

Plane Geometry questions make up about 23% of the total number of questions on the ACT Mathematics Test. The questions test plane geometry concepts such as:

1. Properties and Relations of Plane Figures
    a. Triangles
    b. Circles
    c. Rectangles
    d. Parallelograms
    e. Trapezoids
2. Angles, Parallel Lines, and Perpendicular Lines
3. Perimeter, Area, and Volume

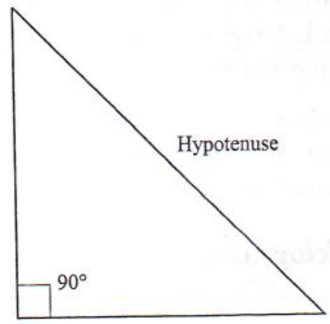
Properties and Relations of Plane Figures

Triangles

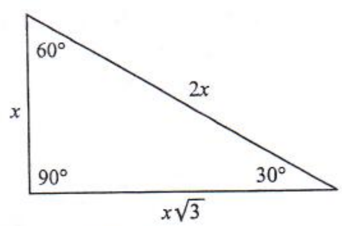


A triangle is a polygon with three sides and three angles. If the measure of all three angles in the triangle are the same and all three sides of the triangle are the same length, then the triangle is an *equilateral* triangle. If the measure of two of the angles and two of the sides of the triangle are the same, then the triangle is an *isosceles* triangle.

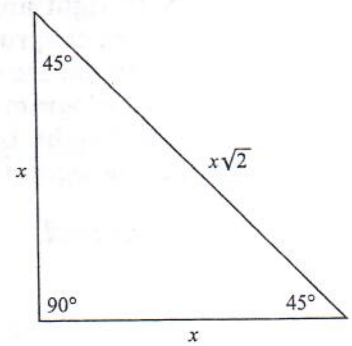
The sum of the interior angles in a triangle is always 180°. If the measure of one of the angles in the triangle is 90° (a right angle), then the triangle is a right triangle, as shown below.



Some right triangles have unique relationships between the angles and the lengths of the sides. These are called *special right triangles*. It may be helpful to remember the following information:



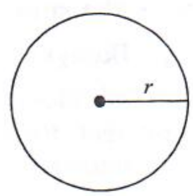
30°–60°–90° triangle



45°–45°–90° triangle

The perimeter of a triangle is the sum of the lengths of the sides. The area of a triangle is $A = \frac{1}{2}$(base)(height). For any right triangle, the Pythagorean Theorem states that $a^2 + b^2 = c^2$, where a and b are legs (sides) and c is the hypotenuse.

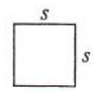
Circles



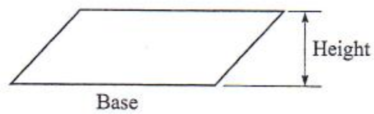
The equation of a circle centered at the point (h, k) is $(x-h)^2 + (y-k)^2 = r^2$, where r is the radius of the circle. The radius of a circle is the distance from the center of the circle to any point on the circle. The diameter of a circle is twice the radius. The formula for the circumference of a circle is $C = 2\pi r$, and the formula for the area of a circle is $A = \pi r^2$.

Rectangles

A rectangle is a polygon with two pairs of congruent, parallel sides and four right angles. The sum of the angles in a rectangle is always 360°. The perimeter of a rectangle is $P = 2l + 2w$, where l is the length and w is the width. The area of a rectangle is $A = lw$. The lengths of the diagonals of a rectangle are congruent, or equal. A square is a special rectangle where all four sides are of equal length, as shown here:



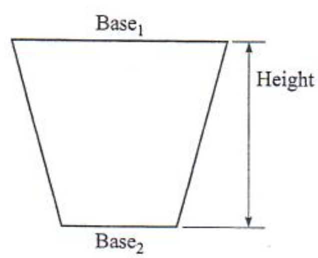
Parallelograms



A parallelogram is a polygon with four sides and four angles that are NOT right angles. A parallelogram has two sets of congruent sides and two sets of congruent angles.

Again, the sum of the angles of a parallelogram is 360°. The perimeter of a parallelogram is $P = 2l + 2w$. The area of a parallelogram is $A = \text{(base)(height)}$. The height is the distance from top to bottom. A rhombus is a special parallelogram with four congruent sides.

Trapezoids

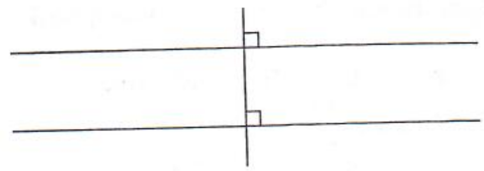


A trapezoid is a polygon with four sides and four angles. The bases of the trapezoid (top and bottom) are never the same length. The sides of the trapezoid can be the same length (isosceles trapezoid), or they may not be. The perimeter of the trapezoid is the sum of the lengths of the sides. The area of a trapezoid is $A = \frac{1}{2}(\text{Base}_1 + \text{Base}_2)(\text{Height})$. Height is the distance between the bases. (The diagonals of an isosceles trapezoid have a unique feature. When the diagonals of a trapezoid intersect, the ratio of the top of the diagonals to the bottom of the diagonals is the same as the ratio of the top base to the bottom base.)

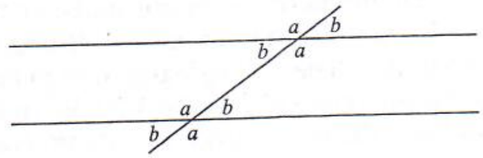
Angles, Parallel Lines, and Perpendicular Lines

Angles can be classified as acute, obtuse, or right. An acute angle is any angle less than 90°. An obtuse angle is any angle that is greater than 90° and less than 180°. A right angle is an angle that is 90°.

When two parallel lines are cut by a perpendicular line, right angles are created, as follows:



When two parallel lines are cut by a transversal, the angles created have special properties. Each of the parallel lines cut by the transversal has four angles surrounding the intersection that are matched in measure and position with a counterpart at the other parallel line. The vertical (opposite) angles are congruent, and the adjacent angles are supplementary; that is, the sum of the two supplementary angles is 180°.



Note: Almost every ACT ever administered has a diagram similar to the one above as part of at least one math question.

Perimeter, Area, and Volume

These formulas are not provided for you on test day. You should make your best effort to memorize them.

Perimeter

The formulas for calculating the perimeter of shapes that appear on the ACT Mathematics Test are as follows:

Triangle: Sum of the Sides
Rectangle and Parallelogram: $2l + 2w$
Square: $4s$ (s is Length of Side)
Trapezoid: Sum of the Sides
Circle (Circumference): $2\pi r$

Area

The formulas for calculating the area of shapes that appear on the ACT Mathematics Test are as follows:

Triangle: $\frac{1}{2}$(Base)(Height)
Rectangle and Square: (Length)(Width)
Parallelogram: (Base)(Height)
Trapezoid: $\frac{1}{2}(\text{Base}_1 + \text{Base}_2)$(Height)
Circle: πr^2

Volume

The formulas for calculating the volume of basic three-dimensional shapes that appear on the ACT Mathematics Test are as follows:

Rectangular Box and Cube: (Length)(Width)(Height)
Sphere: $\frac{4}{3}\pi r^3$
Right Circular Cylinder: $\pi r^2 h$ (h is the height)
Right Circular Cone: $\frac{1}{3}\pi r^2 h$ (h is the height)
Prism: (Area of the Base)(Height)

▶ Trigonometry

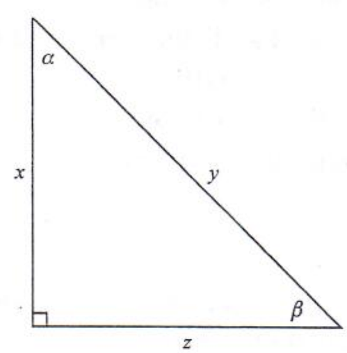
The trigonometry questions make up about 7% of the total number of questions on the ACT Mathematics Test. If you have never taken trigonometry in school, you may still be able to learn enough here to get by on at least a couple of the four questions. (Even if you NEVER learn trigonometry, don't worry; four questions are not likely to seriously affect your score.) The questions test the basic trigonometric ratios (which are related to right triangles, as shown below).

Basic Trigonometric Concepts

The hypotenuse is the side that is opposite the right angle. Sometimes the graph or diagram shown in the question will have the triangle rotated, so make sure that you know where the right angle is and, of course, the hypotenuse, which is directly opposite the right angle.

SOHCAHTOA

SINE (sin) = **O**pposite/**H**ypotenuse (SOH)
COSINE (cos) = **A**djacent/**H**ypotenuse (CAH)
TANGENT (tan) = **O**pposite/**A**djacent (TOA)



$\sin \alpha = \frac{z}{y}$
$\sin \beta = \frac{x}{y}$
$\cos \alpha = \frac{x}{y}$
$\cos \beta = \frac{z}{y}$
$\tan \alpha = \frac{z}{x}$
$\tan \beta = \frac{x}{z}$

Advanced Trigonometric Concepts

Note: The following information will be extremely confusing and intimidating for anyone who has never heard of it before. This information is included only as a review for those readers who have had a trigonometry class. The rest of you should just guess on the two or three questions that might include these concepts.

The secant, cosecant, and cotangent can be found as follows:

$$\text{SEC(secant)} = \frac{1}{\text{COS}} \quad \text{CSC(cosecant)} = \frac{1}{\text{SIN}} \quad \text{COT(cotangent)} = \frac{1}{\text{TAN}}$$

Remember the following Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

Remember the following Trigonometric Identities:

$$\sin(-\theta) = -\sin\theta \quad \cos(-\theta) = \cos\theta \quad \tan(-\theta) = -\tan\theta$$
$$\csc(-\theta) = -\csc\theta \quad \sec(-\theta) = \sec(\theta) \quad \cot(-\theta) = -\cot\theta$$

$$\left.\begin{aligned} \sin(\alpha+\beta) &= \sin\alpha\cos\beta + \sin\beta\cos\alpha \\ \sin(\alpha-\beta) &= \sin\alpha\cos\beta - \sin\beta\cos\alpha \\ \cos(\alpha+\beta) &= \cos\alpha\cos\beta - \sin\alpha\sin\beta \\ \cos(\alpha-\beta) &= \cos\alpha\cos\beta + \sin\alpha\sin\beta \end{aligned}\right\}\text{Addition and Subtraction Formulas}$$

$$\left.\begin{aligned} \sin 2\theta &= 2\sin\theta\cos\theta \\ \cos 2\theta &= \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1 \end{aligned}\right\}\text{Double-angle Formulas}$$

$$\left.\sin^2\theta = \frac{(1-\cos^2\theta)}{2} \qquad \cos^2\theta = \frac{(1+\cos^2\theta)}{2}\right\}\text{Half-angle Formulas}$$

Radians

To change from degrees to radians, multiply the number of degrees by $\frac{\pi}{180}$. For example, 120° is $\frac{120\pi}{180}$ $\frac{2\pi}{3}$ radians. Conversely, to change from radians to degrees, multiply the number of radians by $\frac{180}{\pi}$.